

Exercise: Exponential and Poisson

XDASI Fall 2021

10/21/2021

Example: Lizard mortality

For some animals, like lizards (or hydra), their mortality rate is independent of their age. Over an extended trip to Costa Rica, you studied a large population of lizards and found that their rate of mortality is $\lambda = 0.05$ deaths per month.

⇒ *What is the expected number of deaths per year?*

```
# Poisson: E(X) = SD(X) = lambda = 0.05 per month  
0.05 * 12
```

```
## [1] 0.6
```

⇒ *What is the expected lifespan of one of these lizards?* (In other words, this is the typical “wait time” until one death occurs?)

```
# Exponential: E(X) = SD(X) = 1/lambda = 20 months  
1/0.05
```

```
## [1] 20
```

⇒ *What is the probability an animal will survive more than 20 months?*

Hint:

- This is given by the the exponential survivorship function.

$$P_{exp}(X > 20) = e^{-0.05 \cdot 20} = 0.3679$$

+ Alternatively, it is given by the lower-tail Poisson probability for $\mu = \lambda * t = 0.05 * 20$:

$$P_{pois}(X = 0) = e^{-\lambda t} \frac{\lambda t^x}{x!} = e^{-\mu} \frac{\mu^x}{x!}$$

```
# with exponential
```

```
exp(-0.05*20)
```

```
# manual calculation
```

```
## [1] 0.3678794
```

```
pexp(20, rate = 0.05, lower.tail=F) # with CDF
```

```
## [1] 0.3678794
```

```
# with Poisson: need to multiply units out to mu = lambda * t = 0.05/month * 20 months => 1 new unit
```

```
mu = 0.05*20 # number of deaths in t=20 months
ppois(0, lambda = mu, lower.tail=T)
## [1] 0.3678794
dpois(0, lambda = mu) # same, since just one value for x
## [1] 0.3678794
```

⇒ *What is the probability an animal will survive 20 months or less?*

Hint:

- This is given by the lower-tail CDF of the exponential distribution
- This is equivalent to the upper-tail CDF of the Poisson distribution.

$$P(X \leq 20) = 1 - e^{-0.05*20} = 1 - 0.3679 = 0.6321$$

```
1-exp(-0.05*20) # manual calculation
## [1] 0.6321206
pexp(20,0.05) # with CDF
## [1] 0.6321206

ppois(0, lambda = mu, lower.tail=F) # P(no death > 20 months)
## [1] 0.6321206
```

⇒ *What is the probability an animal will survive between 10 and 15 months?*

Hint:

- To get the total probability across an interval, we subtract the total probability of the lower value from that of the higher value.
- We can't answer this with Poisson, since can only vary # events (not time)!

$$P(10 \leq X \leq 15) = (1 - e^{-0.05*15}) - (1 - e^{-0.05*10}) = e^{-0.05*10} - e^{-0.05*15} = 0.165$$

```
exp(-0.05*10) - exp(-0.05*15) # manual calculation
## [1] 0.1341641
pexp(15,0.05) - pexp(10,0.05) # with CDF
## [1] 0.1341641
```